1.

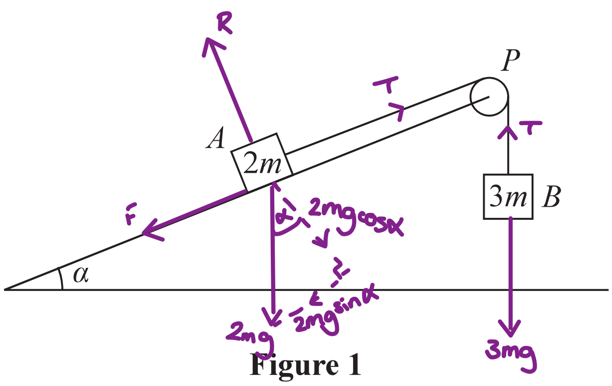


Figure 1

Two blocks, A and B, of masses $2m$ and $3m$ respectively, are attached to the ends of a light string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined at angle α to the horizontal ground, where $\tan\alpha = \frac{5}{12}$

The string passes over a small smooth pulley, P, fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane. Block B hangs freely below P, as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{2}{3}$ $\quad F = \mu R$ $\quad F = \frac{2}{3}R$

The blocks are released from rest with the string taut and A moves up the plane.

The tension in the string immediately after the blocks are released is T.

The blocks are modelled as particles and the string is modelled as being inextensible.

(a) Show that $T = \frac{12mg}{5}$

(8)

After B reaches the ground, A continues to move up the plane until it comes to rest before reaching P.

(b) Determine whether A will remain at rest, carefully justifying your answer.

(2)

(c) Suggest two refinements to the model that would make it more realistic.

(2)

a) $F = ma$

$R(\nwarrow)$:

$R - 2mg\cos\alpha = 0$

$R = 2mg\cos\alpha$ - 1

For A: $R(\nearrow)$

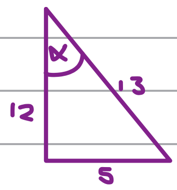
$T - F - 2mg\sin\alpha = 2ma$ - 1

substitute in

$F = \mu R$

$F = \frac{2}{3}R$

$F = \frac{2}{3}(2mg\cos\alpha)$



$\tan\alpha = \frac{5}{12}$

$\cos\alpha = \frac{12}{13}$

$\sin\alpha = \frac{5}{13}$

a) $T - \frac{2}{3}(2mg\cos\alpha) - 2mg\sin\alpha = 2ma$

$T - \frac{2}{3}\left(\frac{24}{13}mg\right) - \frac{10}{13}mg = 2ma$ - ②

$T - 2mg = 2ma$ - ①

For B $R(\downarrow)$

$3mg - T = 3ma$ - ②

$a = \frac{3mg - T}{3m}$ - ②

② into ①

$T - 2mg = 2m \times \left(\frac{3mg - T}{3m}\right)$ - ①

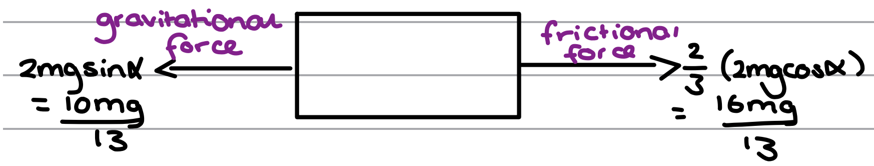
$T - 2mg = \frac{6mg - 2T}{3}$

$3T - 6mg = 6mg - 2T$

$5T = 12mg$

$T = \frac{12mg}{5}$ - ①

b) Forces on A:



$\frac{16mg}{13} > \frac{10mg}{13}$ - ①

∴ frictional force > gravitational force, so A will remain at rest. - ①

c) • It's unlikely that the pulley is smooth - modelling it as rough would be more realistic - ①

• It's also not likely the string is light - it must have some mass. - ①

2. A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$.

A particle of mass m is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is μ.

The particle moves up the plane with a constant deceleration of $\frac{4}{5}g$.

(a) Find the value of μ.

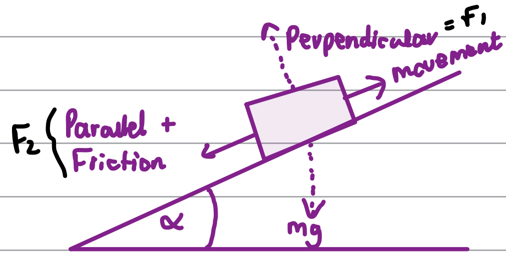
(6)

The particle comes to rest at the point A on the plane.

(b) Determine whether the particle will remain at A, carefully justifying your answer.

(2)

a) μ : Coefficient of Friction

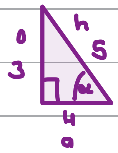


$\tan\alpha = \frac{3}{4}$

$\cos\alpha = \frac{4}{5}$

$\sin\alpha = 3/5$

SOHCAHTOA



$F_1 = mg\cos\alpha$ (1)

$F_1 = \frac{4}{5}mg$

F_2 = Friction + Parallel Force (1)

$F_2 = \mu F_1 + mg\sin\alpha$ (1)

$F_2 = \mu\left(\frac{4}{5}mg\right) + \frac{3}{5}mg$

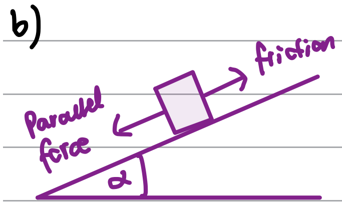
$\Rightarrow \frac{4}{5}mg = \mu\left(\frac{4}{5}mg\right) + \frac{3}{5}mg$ (1) (mg cancels out)

$\Rightarrow \frac{4}{5} = \frac{4}{5}\mu + \frac{3}{5}$

(1)

$\Rightarrow \frac{4}{5}\mu = \frac{4}{5} - \frac{3}{5} = \frac{1}{5} \Rightarrow \mu = \frac{1}{5} \times \frac{5}{4} \Rightarrow \underline{\mu = \frac{1}{4}}$ (1)

b)



At rest friction will be in the upward direction

Friction $= \mu mg\cos\alpha = \frac{1}{4}\cdot\frac{4}{5}mg = \underline{\frac{1}{5}mg}$

Parallel Force $= mg\sin\alpha = \underline{\frac{3}{5}mg}$

$\frac{3}{5}mg > \frac{1}{5}mg$ (1), Parallel force is greater than the frictional force, so then the particle will start to move down the slope and it will no longer remain at A. (1)

3.

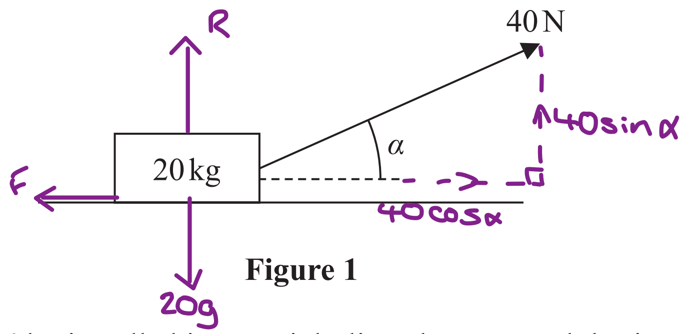


Figure 1

A wooden crate of mass 20 kg is pulled in a straight line along a rough horizontal floor using a handle attached to the crate.

The handle is inclined at an angle α to the floor, as shown in Figure 1, where $\tan\alpha = \frac{3}{4}$

The tension in the handle is 40 N.
The coefficient of friction between the crate and the floor is 0.14
The crate is modelled as a particle and the handle is modelled as a light rod.

Using the model,

(a) find the acceleration of the crate.

(6)

The crate is now pushed along the same floor using the handle. The handle is again inclined at the same angle α to the floor, and the thrust in the handle is 40 N as shown in Figure 2 below.

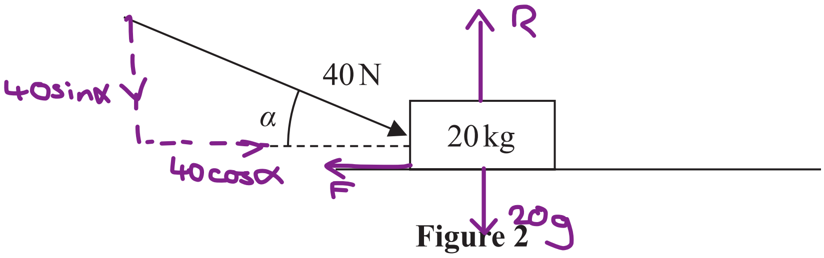


Figure 2

(b) Explain briefly why the acceleration of the crate would now be less than the acceleration of the crate found in part (a).

(2)

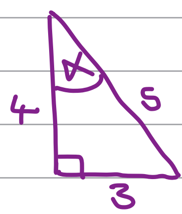
a) $R(\uparrow)$:

$F = ma$

$R + 40\sin\alpha - 20g = 0$ - 2

$R + 24 - 20g = 0$

$R = 172\text{ N}$



$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

$F = \mu R$

$F = 0.14 \times 172$ - 1

$F = 24.08\text{ N}$

a) R(->):

F = ma

$40\cos\alpha - F = 20a$ - 2

$32 - 24.08 = 20a$

$20a = 7.92$

$a = 0.396 \text{ ms}^{-2}$ - 1

b) Pushing would cause the reaction force, R, to increase (as the vertical component of thrust now acts downwards). This will cause the frictional force, F, to increase and so the acceleration will decrease. - 1 1

4. A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

$\tan\alpha = \frac{o}{a}$

A brick P of mass m is placed on the plane.

The coefficient of friction between P and the plane is μ

Brick P is in equilibrium and on the point of sliding down the plane. $\rightarrow F = \mu R$

Brick P is modelled as a particle.

Using the model,

(a) find, in terms of m and g, the magnitude of the normal reaction of the plane on brick P **(2)**

(b) show that $\mu = \frac{3}{4}$ **(4)**

For parts (c) and (d), you are not required to do any further calculations.

Brick P is now removed from the plane and a much heavier brick Q is placed on the plane.

The coefficient of friction between Q and the plane is also $\frac{3}{4}$

(c) Explain briefly why brick Q will remain at rest on the plane. **(1)**

↳ limiting Friction $\Rightarrow F = \mu R$

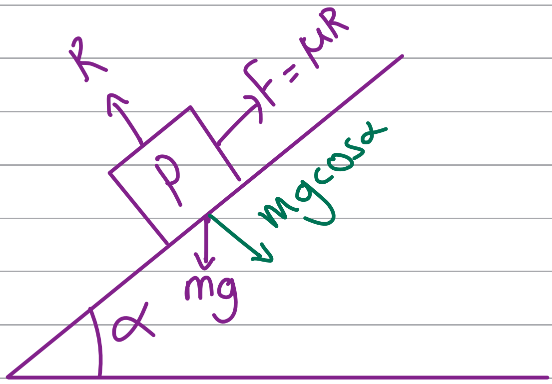
Brick Q is now projected with speed $0.5\,\text{m s}^{-1}$ down a line of greatest slope of the plane.

Brick Q is modelled as a particle.

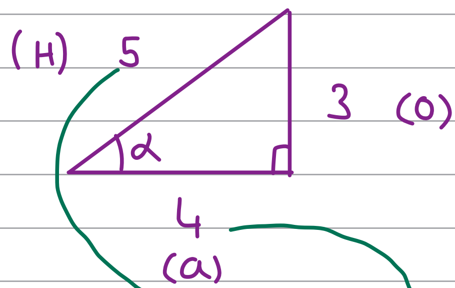
Using the model,

(d) describe the motion of brick Q, giving a reason for your answer. **(2)**

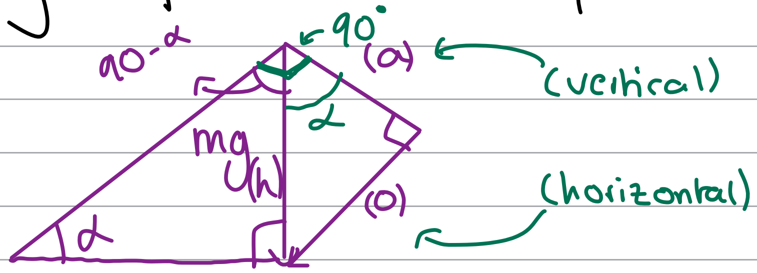
a)



F = friction



Resolving Perpendicular to the plane:



$\cos\alpha = \frac{a}{h}$

$= \frac{4}{5}$

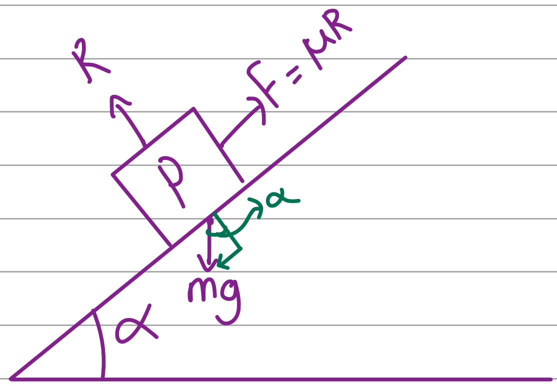
$\cos\alpha = \frac{a}{h} \Rightarrow a = h\cos\alpha$

Vertical = $mg\cos\alpha$ Vertical component = R

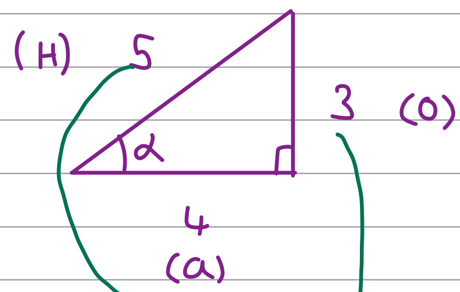
$\therefore R = mg\cos\alpha$

$\therefore R = \frac{4}{5}mg$ using $\cos\alpha = \frac{4}{5}$

b)



F: friction



$R = \frac{4}{5} mg$

$\sin\alpha = \frac{O}{h}$

$= \frac{3}{5}$

Resolving parallel to the plane:

Horizontal component $= mg\sin\alpha$

$F = mg\sin\alpha$

$\mu R = mg\sin\alpha$

$\mu \times \frac{4}{5} \cancel{mg} = \cancel{mg}\sin\alpha$

$\mu = \frac{5}{4} \times \sin\alpha$

$\sin\alpha = \frac{3}{5}$

$\mu = \frac{\cancel{5}}{4} \times \frac{3}{\cancel{5}}$

$\mu = \frac{3}{4}$ as required.

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c)

$$F = \mu R$$ $R = mg\cos\alpha$

$$F = \mu mg\cos\alpha$$ $mg = W$ (weight)

$$F = W \times \mu\cos\alpha$$ $k = \mu\cos\alpha$ (constant)

$$F = kW$$

$$F \propto W$$ (proportional)

Friction is proportional to the weight component.

Friction will increase by the same proportion as the weight component ✓

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d)

Brick Q has no resultant force down the plane.

No resultant force means no acceleration ($F=ma$)

Therefore, brick Q slides down the plane with constant speed.